Estimating Both Supply and Demand Elasticities Using Variation in a Single Tax Rate with General Equilibrium Spillovers

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October 1, 2021

Abstract

Zoutman, Gavrilova, & Hopland (Econometrica, 2018) show that by knowing on 'which side of the market' an 'exogenous' tax is levied one can use a single tax instrument to estimate both a supply and a demand elasticity. This seemingly goes against the intuition that one needs two instruments for two parameters; i.e., a 'supply' and a 'demand' instrument.

I show that the result is only true with partial equilibrium assumptions. Without further assumptions, tax reform induced general equilibrium price spillover effects imply that the tax rates are correlated with the unobserved structural errors. Thus, tax rates on their own are invalid instruments for at least one of the parameters. However, I show that if one can calculate a measure of spillovers, then one can still estimate the two elasticities using one tax reform, but with the spillover measure as an additional instrument.

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1 Introduction

Zoutman, Gavrilova, & Hopland (2018) [ZGH] show that knowing 'which side of the market' a tax is levied allows one to identify both supply and demand elasticities using a single "exogenous" tax instrument, and advocate estimating consumption, housing, and labor elasticities as applications of the method. Citing ZGH, Bibler, Teltser and Tremblay (2018) estimate Airbnb demand and tax evasion, Fajgelbaum, Goldberg, Kennedy and Khandelwal (2020) estimate tariff rate pass through from the 2018 US 'trade war,' and Buettner and Madzharova (2021) estimate price incidence of European VAT reform.

I suggest that researchers consider whether the tax variation has the potential to create 'spillover effects' that cause ZGH approach to be biased. Specifically, I describe how the assumption of "exogenous tax variation" implies partial equilibrium tax incidence, how heterogeneous spillovers cause tax rates to be invalid instruments, and how one can use spillovers, if quantifiable, as identifying variation.[1] Thus, I show one can use a single tax *reform* to identify two elasticities, but not necessarily a single tax rate.

Consider a tax reform that directly affects all tax rates in a labor market. The partial equilibrium [PE] effect only considers the direct quantity and price responses, holding other endogenous variables fixed; whereas, the general equilibrium [GE] allows all market quantities and prices to adjust. I denote 'spillovers' as the additional adjustment to quantities and prices in all markets beyond the PE effect. I assert that spillover effects will be correlated with the tax treatment and heterogeneous across markets, which can be microfounded as in Agrawal and Hoyt (2019) and Watson (2020) for the goods and factor market, respectively.

GE spillovers induce correlation of the tax treatment with unobservable demand or supply shocks causing the ZGH IV approach to be biased for the elasticity on the side facing spillovers. In a Simultaneous Equation Model [SEM] the first stage error term features the structural equations' errors, so 'exogeneity of tax' in the structural equation implies 'exogeneity' in the first stage. Exogeneity implies spillovers must be uncorrelated with treatment which contradicts the assertion. Thus, the tax rates are now invalid instruments. However, if the researcher can find a variable correlated with spillover effects, then this can provide variation to identify a second parameter.

I present monte carlo evidence of spillover bias using a labor market model with imperfectly substitutable labor types and type-specific tax shocks. I find that with log-linear labor supply ZGH is biased for the demand elasticity; with logit labor supply ZGH is biased for both elasticities because the tax rates are invalid instruments. In contrast, my approach is either unbiased or less biased than ZGH in all cases.

[1] ZGH discuss salience, tax avoidance, and lack of pass-through as empirical challenges but not spillover based confounding given their implicit partial equilibrium assumption.

2 Background: ZGH 2018

ZGH use a general price and quantity relationship, but I specify a labor market setting to ground ideas. Begin with the following simultaneous equations model:

$$l_{it}^D = \alpha_0 + \alpha_1 w_{it} + u_{it}^D \qquad l_{it}^S = \beta_0 + \beta_1 w_{it} + \beta_1 \tau_{it} + u_{it}^S \qquad l_{it}^S = l_{it}^D, \tag{1}$$

where all variables are in logs and $\ln[(1+\tau)] \approx \tau$. Labor demand depends on gross wages while supply depends on net wages, which satisfies what ZGH call the 'Ramsey Exclusion Restriction.' ZGH allow for control variables, which I suppress, and they show that their result extends to multi-product markets as long as each good's tax rate has *some* independent variation.

Proposition 1. *ZGH (2018)*
If τ is exogenous with the above SEM, then $\frac{\widehat{\mathsf{Cov}}(l,\tau)}{\widehat{\mathsf{Cov}}(w,\tau)} \to_p \alpha_1$ and $\frac{\widehat{\mathsf{Cov}}(l,\tau)}{\widehat{\mathsf{Cov}}(w+\tau,\tau)} \to_p \beta_1$, where 'exogenous' means that $\mathsf{Cov}(\tau, u^D) = 0$ and $\mathsf{Cov}(\tau, u^S) = 0$.

Proof:
The argument can be seen using the 'first stage' [FS] and 'reduced form' [RF] of the models. The FS is found by equating (1) and (2) and then solving for w, and the RF by substituting the FS into either (1) or (2):

$$w_{it} = \frac{\alpha_0 - \beta_0}{\beta_1 - \alpha_1} + \frac{-\beta_1}{\beta_1 - \alpha_1}\tau_{it} + \frac{u^D - u^S}{\beta_1 - \alpha_1} := \pi_0 + \pi_1 \tau_{it} + v_{it}^w, \tag{2}$$

$$l_{it} = \frac{\alpha_0\beta_1 - \alpha_1\beta_0}{\beta_1 - \alpha 1} + \frac{-\alpha_1\beta_1}{\beta_1 - \alpha_1}\tau_{it} + \frac{\beta_1 u^D - \alpha_1 u^S}{\beta_1 - \alpha_1} := \mu_0 + \mu_1 \tau_{it} + v_{it}^L. \tag{3}$$

The Wald / 2SLS estimator is based on the following covariances:

$$\frac{\mathsf{Cov}(l,\tau)}{\mathsf{Cov}(w,\tau)} = \frac{\mathsf{Cov}(\mu_0 + \mu_1\tau + v^L, \tau)}{\mathsf{Cov}(\pi_0 + \pi_1\tau + v^w, \tau)} = \frac{\mu_1 \mathsf{Var}(\tau) + \mathsf{Cov}(v^L,\tau)}{\pi_1 \mathsf{Var}(\tau) + \mathsf{Cov}(v^w,\tau)}, \tag{4}$$

$$\frac{\mathsf{Cov}(l,\tau)}{\mathsf{Cov}([w+\tau],\tau)} = \frac{\mathsf{Cov}(\mu_0 + \mu_1\tau + v^L, \tau)}{\mathsf{Cov}(\pi_0 + (\pi_1+1)\tau + v^w, \tau)} = \frac{\mu_1 \mathsf{Var}(\tau) + \mathsf{Cov}(v^L,\tau)}{(\pi_1+1) \mathsf{Var}(\tau) + \mathsf{Cov}(v^w,\tau)}. \tag{5}$$

A sufficient condition for the Wald / 2SLS estimators to consistently estimate both structural parameters is that $\mathsf{Cov}(v^L,\tau) = 0$ and $\mathsf{Cov}(v^w,\tau) = 0$ which implies the necessary exogeneity condition in the proposition. Thus, as claimed, 'exogeneity of tax' in the structural equation implies 'exogeneity' of the first stage.

3 Implication of Exogeneity

Consider a labor market where different labor types are imperfectly substitutable in the production function.[2] For example, suppose there are routine and abstract workers, each with a labor demand and supply schedule. Suppose a tax reform for routine workers induces a shift in their labor supply. If firms are able to readjust their labor bundles in response to changes in workers' marginal products, then, depending on the reform's direct incentive effects, firms will demand more or less labor from every skill type relative to PE allocation. Call this demand change a spillover effect that is heterogeneous for each type of worker – see Watson (2020) for further discussion and application to the Earned Income Tax Credit.[3] Thus, one would get the following 'incidence equation':

$$\underbrace{\mathsf{d}w_{it}}_{\text{Wage Change in Data}} = \underbrace{\gamma_1 \mathsf{d}\tau_{it} + \gamma_2 \mathsf{d}Z_{it}}_{\text{Incidence Induced Change}} + \underbrace{\gamma_0 + e_{it}}_{\text{Unobs Wage Change}} \tag{6}$$

where $\mathsf{d}Z$ is a theoretical measurement of the GE spillover effect. I assume that $\mathsf{Cov}(e, \tau) = \mathsf{Cov}(e, Z) = 0$, but I assert that $\mathsf{Cov}(\tau, Z) \neq 0$.

3.1 Reconciling the First Stage and Incidence Equations

To reconcile the two equations, the following equivalence must hold:

$$\underbrace{\gamma_0 + e + \gamma_1 \mathsf{d}\tau + \gamma_2 \mathsf{d}Z}_{\text{Incidence + Unobs}} = \underbrace{\mathsf{d}w}_{\text{Data}} = \underbrace{\frac{\alpha_0 - \beta_0}{\beta_1 - \alpha_1} + \frac{-\beta_1}{\beta_1 - \alpha_1}\mathsf{d}\tau_{it} + \frac{\mathsf{d}u^D - \mathsf{d}u^S}{\beta_1 - \alpha_1}}_{\text{SEM}} \tag{7}$$

One obvious way to reconcile the two equations is the following:

$$e = \frac{-1}{\beta_1 - \alpha 1}\mathsf{d}u^S \qquad \gamma_2 \mathsf{d}Z = \frac{1}{\beta_1 - \alpha 1}\mathsf{d}u^D \qquad \gamma_0 = \frac{\alpha_0 - \beta_0}{\beta_1 - \alpha_1} \qquad \gamma_1 = \frac{-\beta_1}{\beta_1 - \alpha_1}, \tag{8}$$

where $\gamma_2 \mathsf{d}Z$ reflects the demand schedule shift — a demand 'shock.' One can allow for additional demand unobservable wage variation as long as the tax change and spillover measures are uncorrelated; e.g., $e = a_1 \mathsf{d}u^S + a_2 \mathsf{d}u_x^D$ with $\mathsf{Cov}(u_x^D, \tau) = \mathsf{Cov}(u_x^D, Z) = 0$.

Proposition 2.
If there exist demand spillovers Z with $u^D = f(Z)$ and treatment is correlated with spillovers $\mathsf{Cov}(\tau, Z) \neq 0$, then $\mathsf{Cov}(\tau, u^D) \neq 0$, so the ZGH method is inconsistent as tax changes are invalid instruments.

[2]If the types are perfect complements/substitutes, then the tax incidence is not shared, so the first stage is unidentified (as ZGH note) and there are no spillovers.

[3]In Watson (2020), an EITC expansion increases low wage labor supply $\rightarrow$ increases the marginal product of untreated high wage workers $\rightarrow$ shifts high wage demand $\rightarrow$ increases the marginal product of the low wage group; this feedback eventually settles at a new GE allocation.

Two remarks about this proposition. First, ZGH mention that exogeneity may only be conditional on a vector of covariates, x. Alternatively, if spillovers are homogeneous, then covariances are zero, so ZGH follows. Without *ex ante* knowledge of the spillover's functional form, the ZGH method will be inconsistent for reforms that generate GE responses with unspecified heterogeneous spillover effects. The solution is to either find groups that face the same spillovers but different treatment assignment or the same assignment but different spillover effects.

Second, despite the fact that τ is correlated with both v^L and v^W, making τ an invalid instrument in the technical sense, in this linear example the Wald / 2SLS estimator for the supply elasticity is consistent: $\frac{\mathsf{Cov}(l,\tau)}{\mathsf{Cov}(w+\tau,\tau)} = \frac{-\alpha_1\beta_1\,\mathsf{Var}(\tau)+\beta_1\,\mathsf{Cov}(\tau,u^D)}{-\alpha_1\,\mathsf{Var}(\tau)+\mathsf{Cov}(\tau,u^D)} = \beta_1$. With non-linear structural equations–e.g., 'logit' labor supply–spillovers bias the average marginal LATE coefficient from the PE case (Mogstad and Wiswall, 2010; Lochner and Moretti, 2015).

4 Using Spillovers to Estimate Elasticities

Using the RF equation, note the following:

$$\frac{\partial l}{\partial Z} = \frac{\beta_1}{\beta_1-\alpha_1}\frac{\partial u^D}{\partial Z} \quad \& \quad \frac{\partial w}{\partial Z} = \frac{1}{\beta_1-\alpha_1}\frac{\partial u^D}{\partial Z} \quad \Longrightarrow \quad \frac{\partial l/\partial Z}{\partial w/\partial Z} = \beta_1. \tag{9}$$

It is straight-forward to show: $\frac{\partial w}{\partial Z} = \frac{\partial [w+\tau]}{\partial Z}$ and $\frac{\partial l/\partial u^S}{\partial w/\partial u^S} = \alpha_1$. As per intuition, one needs 'demand instruments' for supply parameters and needs 'supply instruments' for demand parameters.

In the context of the labor market SEM, one can use the tax reform treatment as a supply shifter and a measure of spillovers as a demand shifter. Let $\dot{y}_x$ be the residual from from a regression of y on x.

Proposition 3.
If τ is exogenous with the above SEM, then $\frac{\widehat{\mathsf{Cov}}(\dot{l}_\tau,\dot{Z}_\tau)}{\widehat{\mathsf{Cov}}(\dot{w}_\tau,\dot{Z}_\tau)} \to_p \beta_1$ *and* $\frac{\widehat{\mathsf{Cov}}(\dot{l}_Z,\dot{\tau}_Z)}{\widehat{\mathsf{Cov}}(\dot{w}_Z,\dot{\tau}_Z)} \to_p \alpha_1$, *where 'exogenous' means that* $\mathsf{Cov}(\tau,u^S)=0$.

4.1 Empirical Implementation / Simulation Results

Measures for spillovers may come from a structural model or from some *ex ante* knowledge about the nature of spillovers. Watson (2020) uses leave-out averages of close substitute labor groups based on his incidence model, where spillovers are functions of tax changes for other groups. Miguel and Kremer (2004), Clarke (2017), and Huber and

Steinmayr (2021) use distance based measures based on the spatial nature of their data and treatment.

Table 1 reports results from monte carlo simulations using a labor market model where different labor groups are imperfect substitutes. Every labor group has a direct tax shock with independent variation and indirect effects through equilibrium. The GE labor demand elasticity is a function of the elasticity of substitution, ρ, and the tax reform shock.[4] With exponential labor supply: $L_g^S = \delta_g((1+\tau_g)w_g)^\varepsilon$, ZGH is only biased for the demand elasticity. With logit labor supply: $L_g^S = \mathrm{e}^{\beta(1+\tau_g)w_g+\delta_g}/(1+\mathrm{e}^{\beta(1+\tau_g)w_g+\delta_g})$, ZGH is biased for both the demand *and* supply elasticities. For both labor supply models, my approach is either unbiased or has less bias than ZGH. See Appendix A for more details on the model and implementation and discussion of results.

Table 1 – Labor Supply Model Monte Carlo Results

True	$\varepsilon^{Expo}=1,\ \varepsilon^{Logit}=0.88,\ \eta^{\mathsf{Big}}=-0.37,\ \eta^{\mathsf{Small}}=-0.31$							
	Exponential				Logit			
	Big Shock		Small Shock		Big Shock		Small Shock	
	ZGH	Watson	ZGH	Watson	ZGH	Watson	ZGH	Watson
$\hat{\varepsilon}$	1.00	1.00	1.00	1.00	0.48	0.78	0.58	0.96
Bias	0.00	0.01	0.00	0.00	-0.40	-0.10	-0.38	0.08
$\hat{\rho}$	-0.47	-0.35	-0.42	-0.32	-0.39	-0.35	-0.36	-0.31
Bias	-0.10	-0.02	-0.11	-0.01	-0.02	-0.02	-0.05	0.00

Estimates are from a pooled regression of 1,000 simulated markets with fixed effects for labor groups and initial wage ventiles. Standard errors are clustered by labor groups (100 clusters). See Appendix A for more details and discussion of construction, results, and alternative specifications.

5 Conclusion

Zoutman, Gavrilova and Hopland (2018) discuss identification of supply and demand elasticities using a single tax instrument with two assumptions: knowing which side of the market faces and responds to taxes and that taxes are exogenous. I discuss how exogeneity implies an absence of spillovers – i.e., partial equilibrium – and provide an example where spillovers cause the Wald / 2SLS estimate to be inconsistent for at least one of the elasticities. I additionally show that having a measure of spillovers can provide an additional instrument which allows for identification of both elasticities.

[4]See Appendix A for a derivation of the elasticity.

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A Appendix

This appendix describes the simulation underlying Table 1.

A.1 Simulation Details: Theory

The model in the simulation is based on a simplified version of Watson (2020).

Labor Supply I posit two different labor supply functions. The first is an exponential function:

$$L_g^{S,Expo} = \delta_g \left((1+\tau_g)w_g\right)^{\varepsilon}. \tag{10}$$

For this function, I assume all types have the same labor supply elasticity: $\varepsilon_g^S = 1$. The second is a logit function:

$$L_g^{S,Logit} = \frac{\mathrm{e}^{\beta(1+\tau_g)w_g + delta_g}}{1+\mathrm{e}^{\beta(1+\tau_g)w_g + delta_g}}. \tag{11}$$

For this function, I do not force all types to have the same labor supply elasticity, $\varepsilon_g^S = \beta(1+\tau_g)w_g \cdot (1 - L_g^{S,L})$.

Labor Demand There is a single consumption good produced by perfectly competitive industry with a perfectly competitive labor market. I model the production function as CES with constant returns to scale:

$$Q = A \cdot \left(\sum_g \alpha_j L_g^{\frac{1+\rho}{\rho}}\right)^{\frac{\rho}{1+\rho}}, \tag{12}$$

where $g \in G$ index labor groups, $\rho < 0$ is the elasticity of substitution, and $\alpha_g > 0$ and $\sum_j \alpha_g = 1$. Under profit maximization, the firm sets the relative factor quantities are a function of the relative factor prices: $\frac{L_g}{L_{g'}} = \left(\frac{w_g/\alpha_g}{w_{g'}/\alpha_{g'}}\right)^{\rho}$.

Through substitution, I arrive at the conditional factor demand as:

$$L_g^D(w_g; w_{\neg g}, Q) = (Q/A)\alpha_g^{\frac{\rho}{1+\rho}} \left(\frac{\alpha_g(\frac{w_g}{\alpha_g})^{1+\rho}}{\sum_h \alpha_h(\frac{w_h}{\alpha_h})^{1+\rho}}\right)^{\frac{\rho}{1+\rho}} \tag{13}$$

$$= A_g \cdot (\Gamma_g)^{\frac{\rho}{1+\rho}}. \tag{14}$$

Note: $\Gamma_g = (w_g L_g / \sum_h w_h L_h)$ is both the cost share and the output elasticity.

Equilibrium The equilibrium conditions are set by clearing the labor market and forcing zero profits, with output price normalized to one. That is:

$$\textbf{Labor Clear}: \quad \frac{L_g^S}{L_h^S} = \left(\frac{w_g/\alpha_g}{w_h/\alpha_h}\right)^{\rho} \qquad \forall h \in G \setminus g \tag{15}$$

$$\textbf{Zero Profits}: \quad 1 = c(\{w_j\}_{j\in G}, \tag{16}$$

where $c(\cdot)$ is the unit cost function. By Walras' Law, no matter the (well-behaved) utility function and consumption demand, the output market clears.

Equilibrium Labor Demand Elasticity The standard partial equilibrium factor demand elasticity is $\eta_g^{D,PE} = \rho(1-\Gamma_g)$. To find the general equilibrium factor demand elasticity, I calculate:

$$\begin{aligned}
\frac{\partial L_g^D}{\partial w_g}\frac{w_g}{L_g} &= \rho(1-\Gamma_g) + \sum_{h\backslash g} \frac{\partial Q}{\partial L_h}\frac{\partial L_h^S}{\partial w_h}\frac{\partial w_h}{\partial w_g} \mathsf{d}\tau_g && (17)\\
&= \rho(1-\Gamma_g) + \sum_{h\backslash g} \Gamma_h \varepsilon_h^S \frac{-w_g L_g}{w_h L_h} \mathsf{d}\tau_g && (18)\\
&= \rho(1-\Gamma_g) - \mathsf{d}\tau_g \Gamma_g \sum_{h\backslash g} \varepsilon_h^S && (19)\\
&= \rho(1-\Gamma_g) - \mathsf{d}\tau_g \Gamma_g (N-1)\, \mathsf{E}[\varepsilon_h^S] && (20)\\
&:= \eta_g^{D,GE}. && (21)
\end{aligned}$$

Remark. I use the fact that I am shocking the equilibrium by changing the tax rates to calculate this object. Without this, because unit cost must be kept constant, an increase in any one price would be negated in GE. That is, the tax change induced variation in quantities such that even though the unit cost is constant there is still non-trivial variation in the relative prices.

Remark. If $\mathsf{d}\tau_g \approx 0$, then $\eta_g^{D,GE} = \eta_g^{D,PE}$. Further, if $\Gamma_g \approx 0$, then $\eta_g^{D,GE} = \eta_g^{D,PE} = \rho$.

A.2 Simulation Details: Implementation

I implement the above model using the following simulation using the following steps — further details follow.

First, I draw parameters and numerically solve for an initial wage vector that satisfies the equilibrium conditions. Second, I create a tax reform. Third, I solve for the new wages that satisfy the equilibrium where only the tax system has changed. Fourth, I calculate the percent changes in these variables–quantities, wages, tax rates–and create a leave-out-average for the additional demand variation instrument (as generally described in the main text and mechanically below).

I repeat these steps for 1000 'markets' for the following model types $\{\text{Expo,Logit}\} \times \{\text{Big Shock, Small Shock}\}$, which creates four datasets. For each dataset, I do pooled just-identified regressions using the variables of interest to get the monte carlo values. Alternatively, I could split the 1000 'markets' into market-groups (of say 100 or 200), and then either include market-group fixed effects in the pooled regression or do market-group specific regressions and then average the estimated parameters across market-groups. I find that this makes no qualitative difference when assessing mean values and overall bias.

Fixed Parameters - Constant Across Markets I create one thousand markets: $t \in T = \{1, 2, \ldots, 1000\}$. In each market, I set the number of labor groups at $g \in G = \{1, 2, \ldots, 100\}$, where $N = |G|$. I create four types/demographics of workers $d \in D = \{1, 2, 3, 4\}$. Note: the demographic terms only matter for the supply function and the tax system. Note: each labor market group maps to only one demographic group, so I suppress the d subscript when not necessary.

For the Exponential labor supply model, I set $\varepsilon_g^S = 1$ for all $g \in G$. For the Logit labor supply model, I set $\beta_{d,g} = 5$ for all $g \in G$. I set the elasticity of labor substitution at $\rho = -0.3$. I set $\alpha_g = \frac{g-1}{g \cdot N} + \left(\frac{g}{N}\right)^3$, which is a convex function on the domain G.

I set initial tax rates as a non-linear function that depends on labor group and demographics — this mimics progressive taxation with a group based, means-tested tax credit (like the Earned Income Tax Credit). Let $k_1 = (d_g == 1)$, and indicator for belonging to demographic type 1.

$$\tau_g^{\textbf{Init}} = \begin{cases} (-0.0014 \cdot g) - (-0.0014 \cdot g)/2 \cdot k_1 & \text{for } g < 75 \\ (-0.1 - 0.0062 \cdot g) - (-0.1 - 0.0062 \cdot g)/2 \cdot k_1 & \text{for } g \geq 75. \end{cases} \tag{22}$$

This tax system starts at $\tau_1 = 0$, goes linearly to $\tau_{75} = -0.1$ and then linearly but more steeply to $\tau_{100} = -0.25$; however, the tax rate is cut in half if a member of demographic type 1.

Varying Parameters - Change Across Markets The following parameters vary for each 'market' but do not vary within a 'market.' In each market, I draw a total factor productivity term $A_t \sim \mathsf{N}(\mu = 100, \sigma = 10)$. In each market, I draw type specific supply shifters. For the Exponential supply, $\delta_{gt} \sim \mathsf{U}(\min = 1, \max = 2)$. For the Logit supply, $\delta_{gt} \sim \mathsf{N}(\mu = -3, \sigma = 0.2)$.

Only the tax system changes within a 'market.' First, I generate two random shocks correlated with the wage distribution. I draw $v^1_{gt} = (u^1 + 0.05)/w^{\text{Init}}_{gt}$ and $v^2_{gt} = (u^2 + 0.02)/w^{\text{Init}}_{gt}$, where $u^1 \sim \mathsf{U}(0, 0.1)$ and $u^2 \sim \mathsf{U}(0, 0.05)$. Second, I draw a random tax rate shock uncorrelated with wages, $u^3_{gt} \sim \mathsf{U}(0, 0.1)$. Finally, the new tax rate is simply:

$$\tau^{\text{Post, Big}}_{gt} = \tau^{\text{Init}}_{gt} + v^1_{gt} \cdot k_1 + v^2_{gt} \cdot k_4 + u^3_{gt}. \tag{23}$$

For the small shock, I calculate $\tau^{\text{Post,Small}}_{gt} = (\tau^{\text{Post, Big}}_{gt}) \cdot 0.1 + (\tau^{\text{Init}}_{gt}) \cdot 0.9$. This tax shock ensures that each labor group gets some shock and that the shock is correlated with wages and demographics.

Numerical Solution I solve for the N wages using the $N-1$ labor clearing conditions and the single zero profit condition. I implement this by using the equilibrium conditions as non-linear equality constrains using MATLAB's `fmincon` with all tolerances set to $1e^{-13}$. In all four simulation groups, for every market, and for both initial and post-reform solutions, the solver always reaches a unique equilibrium satisfying all constraints.

Regression The for each market the following variables are created:

$$\{L^{\text{Init}}_{gt}, L^{\text{Post}}_{gt}, w^{\text{Init}}_{gt}, w^{\text{Post}}_{gt}, n^{\text{Init}}_{gt}, n^{\text{Post}}_{gt}, \tau^{\text{Init}}_{gt}, \tau^{\text{Post}}_{gt}\},$$

where $n^j_g = (1 + \tau^j_g) \cdot w^j_g$ is the tax-inclusive net-wage. For each, I create percent change variables $\dot{x}_g = \frac{x^{\text{Post}}_g}{x^{\text{Init}}_g} - 1$.

To create a substitution based instrument, I calculate a within-market leave-out-average of the tax shock:

$$Z_g = \frac{\sum_h \dot{\tau}_h - \sum_h \dot{\tau}_h \cdot (\mathsf{q}^w_h == \mathsf{q}^w_g)}{N - \sum_h (\mathsf{q}^w_h == \mathsf{q}^w_g)}, \tag{24}$$

where q^w_h is a quantile indicator based on the initial wage distribution. Table 1 reports values using ventiles (20), so there are five labor types in each ventile. I find that I can use deciles (10 per group) and attain nearly identical results; however, using quintiles (20 per group), I find the instrument lacks sufficient variation given the tax reform that I use.

With the variables, I do the following regressions:

$$\text{ZGH-Supply}: \quad \texttt{ivreghdfe}\ \dot{L}_{gt}\ (\dot{n}_{gt} = \dot{\tau}_{gt}),\ \texttt{absorb}(i.g)\ \texttt{cluster}(g) \tag{25}$$

$$\text{ZGH-Demand}: \quad \texttt{ivreghdfe}\ \dot{L}_{gt}\ (\dot{w}_{gt} = \dot{\tau}_{gt}),\ \texttt{absorb}(i.g)\ \texttt{cluster}(g) \tag{26}$$

$$\text{Watson-Supply}: \quad \texttt{ivreghdfe}\ \dot{L}_{gt}\ \dot{\tau}_{gt}\ (\dot{w}_{gt} = Z_{gt}),\ \texttt{absorb}(i.g)\ \texttt{cluster}(g) \tag{27}$$

$$\text{Watson-Demand}: \quad \texttt{ivreghdfe}\ \dot{L}_{gt}\ Z_{gt}\ (\dot{w}_{gt} = \dot{\tau}_{gt}),\ \texttt{absorb}(i.g)\ \texttt{cluster}(g). \tag{28}$$

These regressions pool across markets and absorb and cluster based on the labor market group. The parameter of interest from these regressions is the coefficient on the endogenous variable, which can be interpreted as either a supply or demand elasticity depending on the specification. As mentioned earlier, I find no qualitative (or even much quantitative) difference in the reported parameters based on splitting the markets into smaller groups and averaging or including market-group fixed effects. However, since there is only tax-reform changes in each market, including a fixed effect for each market t absorbs nearly all of the variation of interest and all models perform poorly.

A.3 Additional Simulation Results

Table 2 reports alternative specifications. I reproduce the baseline estimates in panel A.

Panel B removes the labor group fixed effects to present the ZGH approach in a more favorable setting. Without labor group fixed effects, the logit labor supply elasticity estimates deteriorate substantially for both approaches. The labor demand estimates improve for the ZGH approach but these estimates are still more biased relative to the Watson approach which are not effected. Panel C replaces the labor-group fixed effects for wage-ventile fixed effects, since the IVs are calculated at this level. As the wage ventile FEs are a subset of the labor group FEs (since each labor group maps to just one ventile), this case is like an attenuated version of similar to Panel B.

Panel D replaces wage-ventile based IV for a wage-decile based IV. This has essentially no effect but the logit labor supply elasticities are attenuated slightly. Panel E replaces wage-ventile based IV for a wage-quintile based IV. The logit labor supply elasticities are further attenuated. These panels reduce variation in the constructed instrument, so there the instrument is weaker and estimates tend towards zero.

Panel F splits the 1000 markets into 50 groups of 20 markets and includes a fixed effect for these groups. This has essentially no effect. Panel G splits the 1000 markets into 50 groups of 20 markets, does separate regressions for each group, and then averages the estimates. The logit labor supply estimates suffer because they use spillover variation in the same labor group segment across markets, so by reducing the number of markets, this reduced variation in the instrument.

Overall, these alternative specification illustrate three points. First, the ZGH approach is biased for the labor demand elasticity when there are spillovers regardless of specification. Second, using a demand-spillover based IV for the supply elasticity is still subject to all the issues related to using an IV approach, which can be sensitive to specification — practitioners should use care. Third, when the linearity assumptions of ZGH are not met, then the supply elasticity estimate will also be biased, and this bias can be greater or lesser of the bias in using a weaker demand-spillover IV.

Table 2 – Additional Labor Supply Model Monte Carlo Results

True	$\varepsilon^{Expo} = 1,\ \varepsilon^{Logit} = 0.88,\ \eta^{\mathsf{Big}} = -0.38,\ \eta^{\mathsf{Small}} = -0.31$							
	Exponential				Logit			
	Big Shock		Small Shock		Big Shock		Small Shock	
	ZGH	Watson	ZGH	Watson	ZGH	Watson	ZGH	Watson
	(A) Baseline							
$\hat{\varepsilon}$	1.00	1.00	1.00	1.00	0.48	0.78	0.58	0.96
$\hat{\rho}$	-0.47	-0.35	-0.42	-0.32	-0.39	-0.35	-0.36	-0.31
	(B) No Fixed Effects							
$\hat{\varepsilon}$	1.00	1.02	1.00	1.00	1.33	0.14	1.49	0.25
$\hat{\rho}$	-0.38	-0.35	-0.34	-0.32	-0.35	-0.35	-0.32	-0.31
	(C) Wage-Ventile Fixed Effects							
$\hat{\varepsilon}$	1.00	1.00	1.00	1.00	0.73	0.48	0.82	0.64
$\hat{\rho}$	-0.38	-0.35	-0.35	-0.31	-0.36	-0.35	-0.32	-0.31
	(D) Alt IVs: Decile Based							
$\hat{\varepsilon}$	–	1.01	–	1.00	–	0.76	–	0.93
$\hat{\rho}$	–	-0.36	–	-0.32	–	-0.35	–	-0.31
	(E) Alt IVs: Quintile Based							
$\hat{\varepsilon}$	–	1.02	–	1.00	–	0.70	–	0.86
$\hat{\rho}$	–	-0.37	–	-0.32	–	-0.35	–	-0.31
	(F) Market-Groups FEs							
$\hat{\varepsilon}$	1.00	1.00	1.00	1.00	0.48	0.77	0.58	0.94
$\hat{\rho}$	-0.46	-0.35	-0.42	-0.32	-0.39	-0.35	-0.36	-0.31
	(G) Average Over Market-Groups							
$\hat{\varepsilon}$	1.00	1.00	1.00	1.00	0.48	0.69	0.58	0.84
$\hat{\rho}$	-0.47	-0.35	-0.42	-0.32	-0.39	-0.35	-0.36	-0.31

Estimates are from a pooled regression of 1,000 simulated markets with fixed effects for labor groups and initial wage ventiles. Standard errors are clustered by labor groups (100 clusters). See Appendix A for more details and discussion of construction, results, and alternative specifications.